

# Blade Design Project Report

MAE 4700: Finite Element Analysis in Mechanical and Aerospace Engineering

Due Date: December 15th, 2025

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Sources: N/A

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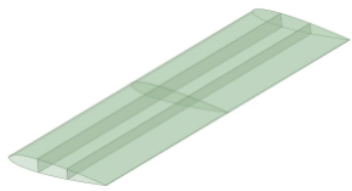
## Experimental Background

### A. Abstract

Finite-element analysis (FEA) is a common method for improving wing blade aerodynamics, especially in complex operating conditions. The following report provides recommendations for optimizing a modern wing blade's performance through a finite-element-based approach in ANSYS. By parameterizing structural dimensions of the blade, such as location, quantity, and thickness of blade components (skin, spar, and ribs), the blade's total weight is reduced by 41.80%. Major improvements were made by increasing the number of ribs from 2 to 15 across the span of the blade, and significantly reducing the overall thickness of the spars and ribs to 0.00585 and 0.00103 m, respectively. With these conditions, the wing blade operated at  $\delta_{Max} \approx 0.37446 \text{ m}$ ,  $\eta \approx 1.6059$ , and  $\sigma_v = 156.92 \text{ MPa}$ , within the design constraints of  $\sigma_v < 252 \text{ MPa}$  and  $\delta_{Max} < 0.375$ . Throughout the report, figures computing the finite-element solutions are provided to further illustrate the optimized design geometry and performance in terms of stress and deformation response.

### B. Introduction

Typical wing blades require optimization through FEA-approaches to streamline decision-making for complex design constraints and automate numerical solutions. As a practice of this design process, an example of a modern wing has been provided by the MAE 4700 course staff that requires weight reduction, given limitations on maximum Von Mises stress and tip deflection. Additionally, large-scale constraints are outlined as general geometric design limitations. This limits the scope of major design improvements to changing skin, spar, and rib attributes. Parameters of interest for experimentation throughout the FEA study include reducing the number of spars and ribs in the wing, thinning the skin, and altering the placement of the various components in the design. The following is a top-down perspective on the major notes from the experimental setup, design constraints, and model parameters:

Wing Design Parameters	
 <p>NACA 0012 Airfoil, 15 m in Length, Aluminum 2024-T36</p> <ol style="list-style-type: none"> <li>1. Mass: <math>m = 13669 \text{ kg}</math></li> <li>2. Volume: <math>V = 4.9226 \text{ m}^3</math></li> <li>3. Surface Area: <math>SA = 164.09 \text{ m}^2</math></li> <li>4. Density: <math>\rho = 2776.8 \text{ kg/m}^3</math></li> </ol>	Young's Modulus: $E = 74 \text{ GPa}$
	Poisson's Ratio: $\nu = 0.3$
	Yield Strength: $\sigma_{Yield} = 378 \text{ MPa}$
	Lower Surface Pressure: $P_{Upper} = 2500 \text{ Pa}$
	Upper Surface Pressure: $P_{Upper} = -6000 \text{ Pa}$
	Skin, Spar, and Rib Thickness: $t_{All} = 1.0 \text{ cm}$
	Factor of Safety: $\eta = 1.5$

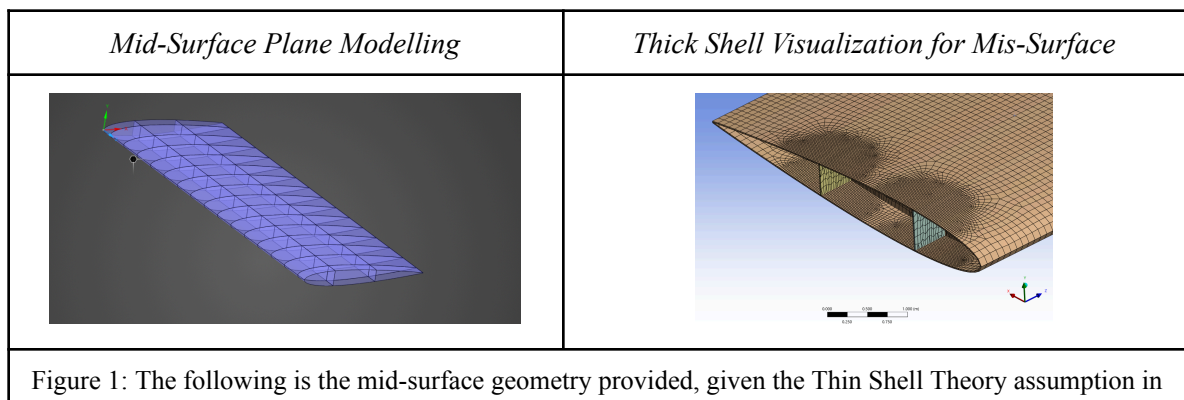
Design Variables (Inputs)	Design Constraints (Outputs)
Skin Thickness	Maximum Von Mises Stress ( $\sigma_V < \frac{\sigma_{Yield}}{\eta} = 252 \text{ MPa}$ )
Spar Thickness	Maximum Tip Deflection ( $\delta_{Max} < 0.375$ ) (2.5% Span)
Rib Thickness	Constant Outer Mold Line
Number/ Location of Spars	Constant Span
Number/ Location of Ribs	Constant Aspect Ratio
Rib Geometry	Constant External Geometry

The aforementioned wing is computed with the static structural software in ANSYS to simulate the wing blade in typical operating conditions (defined by the pressure boundary conditions). To isolate areas of improvement, the mid-surface geometry of the wing blade is made in ANSYS Discovery and altered through the “Parameter Set” feature of the ANSYS Workbench to efficiently compute various output conditions. From each of these simulations, the goal is to evaluate the maximum Von Mises stress and tip deflection, and continuously iterate across different parameters until one frontrunner emerges.

### C. Mathematical Model and Numerical Solution Strategy

#### *Mathematical Model*

Blade wings are considered as a simplified thin-wall structure for computational approaches, thus introducing Thin Shell Theory as the primary governing principle for the numerical strategy. Shell Theory is a mathematical method for describing thin-walled structures as one-dimensional, mid-surfaces without the additional thickness. This approach is specifically valid for our model, considering that the thickness of the thin-walls is much smaller relative to the larger dimensions of the wing, and the thickness is consistent across all the various wall segments of the blade. This methodology is extremely computationally efficient in ANSYS, as by removing the additional elements resulting from the thickness of the thin-wall, the mathematical model is required to solve for far fewer numerical solutions while still preserving the dominant structural response.



the mathematical model, and the visualization of the thin shells in ANSYS through the “Thick Shell” option in the visualizer

In addition to the Thin Shell Theory, the wing blade is governed by the Principle of Minimum Potential Energy, where the configuration or deformation of an elastic system aims to minimize the total potential energy. For our application, this governs the mechanical response of the blade elements and leads the ANSYS model to converge on solutions for deformation with the least potential energy. This principle specifically governs the equilibrium of displacement for all varieties of mechanical systems, from springs to shells, based on the geometric degrees of freedom (DOF).

Both the Thin Shell Theory and the Principle of Minimum Potential Energy are leveraged to compile the mathematical model for the deformation of the wing blade geometry under the specified load. With the wing blade modelled as a mid-surface for simplification, the Principle of Minimum Potential Energy is solved along the simplified surface of the undeformed geometry. Similar to the basic linear spring model, the mathematical model for the shells consists of strain energy and external force parameters that consist of the following:

1. Essential boundary conditions (nodal translations and rotations, known/unknown DOF)
2. Location of the undeformed geometric mid-surface (modelled geometry in ANSYS Discovery)
3. External forces/ surface loads
4. Internal material properties (Young’s Modulus,  $E$ , Moment of Inertia,  $Et^3$ , Poisson’s Ratio,  $\nu$ )
5. Surface Mesh for Discrete Shell Elements

### Numerical Solutions

For the numerical solution in ANSYS, the formulation of this numerical solution is similar to characterizing energy for a linear spring. With the basic mechanical response of a thin-walled structure being similar to a linear spring, our system is mathematically modelled based on the basic structural models for a spring with additional degrees of freedom. A linear spring is considered a single element with 2 DOF, 1 stiffness ( $k$ ), 1 force ( $F$ ), and 2 displacements ( $u_1$  and  $u_2$ ), with the following equations:

$$\Pi_n(u) = \text{Strain Energy} - \text{External Work}$$

$$\Pi_{1, Spring}(u) = \frac{1}{2}k(u_2 - u_1)^2 - Fu_2, \frac{d\Pi_{1, Spring}}{du} = 0$$

For this specific use case, the internal mechanical characteristics ( $k$ ) and the boundary conditions on the element ( $F$  and  $u_1$ ) are required to solve for the displacement  $u_1$  that minimizes the total potential energy.

For structures with higher dimensions (i.e., beams, trusses, and shells), a similar system is implemented to characterize potential energy in terms of DOF, relate to the boundary conditions, and solve for displacements. The following is an approximation of the numerical solution for the ANSYS software:

$$\Pi_{1, Surface}(u) = \sum_{e=1}^{n_{elm.}} \left( \frac{1}{2}q_e^T K_e q_e - q_e^T F_e \right), \frac{d\Pi_{1, Surface}}{dq} = 0$$

where  $q_e$  = Element DOFs,  $K_e$  = Global Stiff. Matrix ( $E$ ,  $I$ , etc.), and  $F_e$  = Ext. Force

Although typical exact solutions require integration over the boundary of the surface, the finite element approach instead sums the discretized elements of the surface as individual linear springs.

The equation is dependent specifically on the degrees of freedom for the respective element. ANSYS automatically calculates DOF for each element with  $DOF_{Elm.} = DOF_{3D} * n_{Nodes}$ , essentially by multiplying the DOF within a typically 3D surface by the number of nodes comprising the surface element. While the former is fixed for the following system (6 DOF with 3 rotational and 3 transverse DOF), the number of nodes for each respective shell element is dependent on the node mesh sizing method. Typically, surface nodes within ANSYS are computed as 4-node surface elements (squares), meaning these elements have 24 DOF according to the ANSYS software. However, multiple sections are refined with adaptive methods to remove singularities and prevent duplications of stress at the same point where mid-surface planes intersect. As such, some elements are not exclusive 4-nodes that require continuous computation by the ANSYS software to compute DOF. With these two methodologies, the software automatically defines meshes that are compatible with the various shell element methodologies and produces a solvable, continuous mesh for the simulation

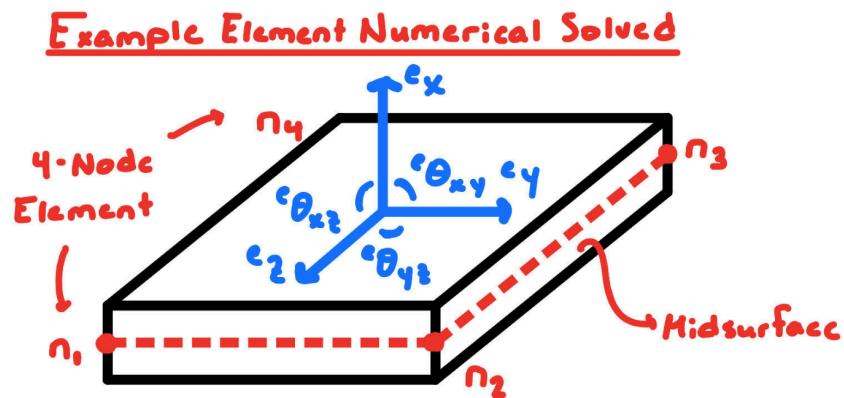
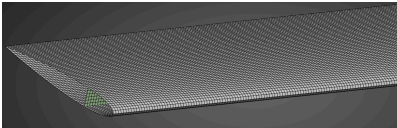
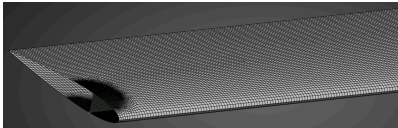
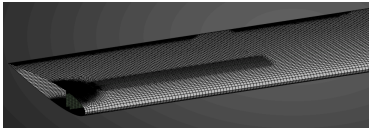
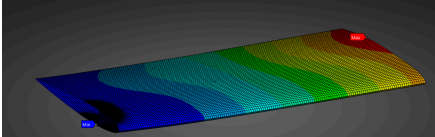
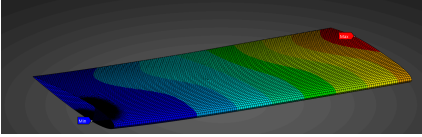
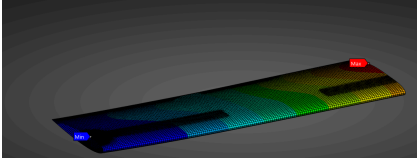
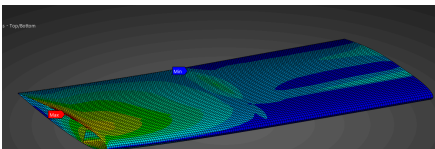
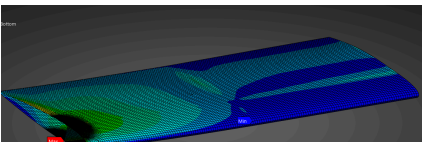
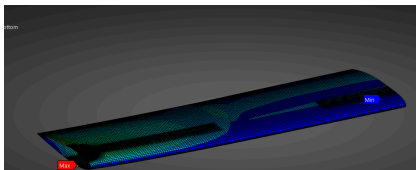
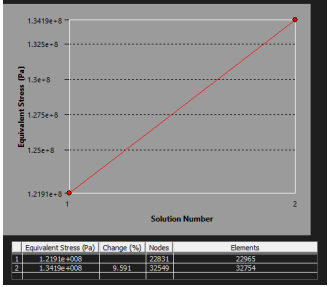


Figure 2: The following is a diagram of an example 4-node shell element computed in the numerical solutions of the ANSYS software

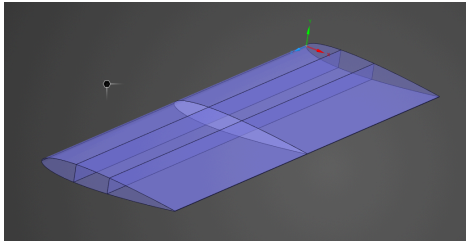
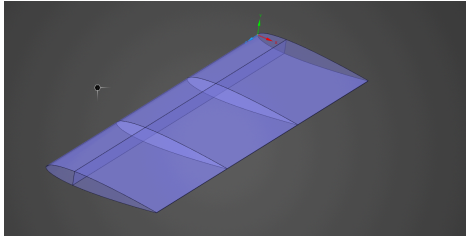
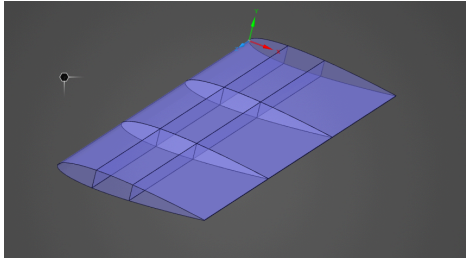
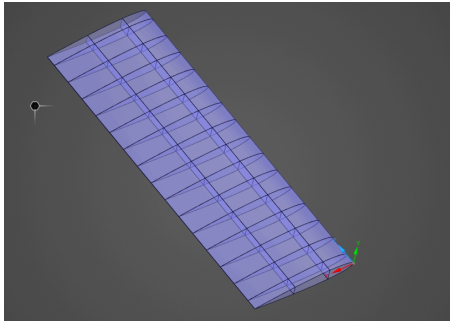
## ANSYS Set-Up and Optimization Procedure

### D. Mesh Refinement

Although ANSYS automatically computed the mesh for the surface elements of the blade design, this process resulted in singularities at the intersection point of mid-surface planes. As previously mentioned, due to interference between the thickness of two mid-surface planes, mechanical responses at these locations are improperly modelled. As such, manual and adaptive mesh refinements were implemented as a method for refining the original mesh provided by the automatic discretization of the mid-surface geometry. The following is the gradual mesh refinement method for improving the elements for the FEA computation, ranging from the original mesh to adaptive refining with the Von Mises stress:

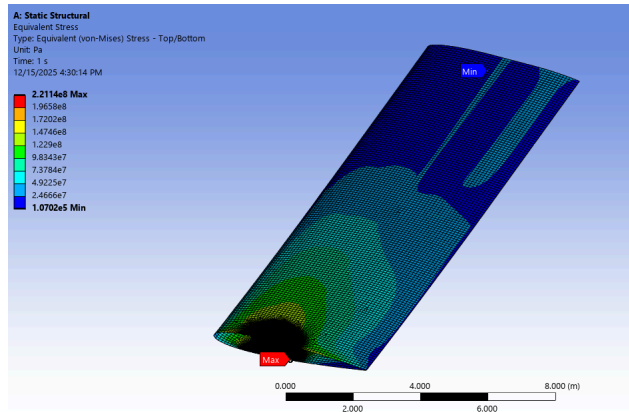
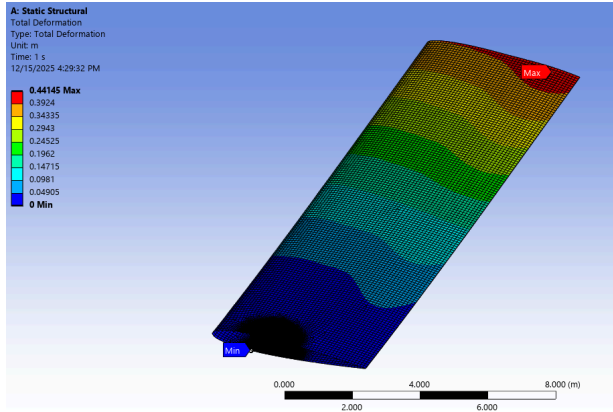
Mesh Varieties (Computed with a Variation of the Initial Geometry)																	
Original Mesh	Manual "Sphere of Influence"	Automated Von Mises Stress															
 <p>21013 Elements, 19834 Nodes</p>	 <p>27833 Elements, 26516 Nodes</p>	 <p>32754 Elements, 31620 Nodes</p>															
 <p><math>\delta_{Max} \approx 0.22753 \text{ m}</math></p>	 <p><math>\delta_{Max} \approx 0.22763 \text{ m}</math></p>	 <p><math>\delta_{Max} \approx 0.22753 \text{ m}</math></p>															
 <p><math>\sigma_{V, Max} = 97.093 \text{ MPa}</math></p>	 <p><math>\sigma_{V, Max} = 121.91 \text{ MPa}</math></p>	  <table border="1"> <thead> <tr> <th>Solution Number</th> <th>Equivalent Stress (Pa)</th> <th>Change (%)</th> <th>Nodes</th> <th>Elements</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1.2191e+08</td> <td></td> <td>27951</td> <td>27955</td> </tr> <tr> <td>2</td> <td>1.3419e+08</td> <td>9.591</td> <td>31599</td> <td>31594</td> </tr> </tbody> </table> <p><math>\sigma_{V, Max} = 143.19 \text{ MPa}</math></p>	Solution Number	Equivalent Stress (Pa)	Change (%)	Nodes	Elements	1	1.2191e+08		27951	27955	2	1.3419e+08	9.591	31599	31594
Solution Number	Equivalent Stress (Pa)	Change (%)	Nodes	Elements													
1	1.2191e+08		27951	27955													
2	1.3419e+08	9.591	31599	31594													

## E. Design Comparisons

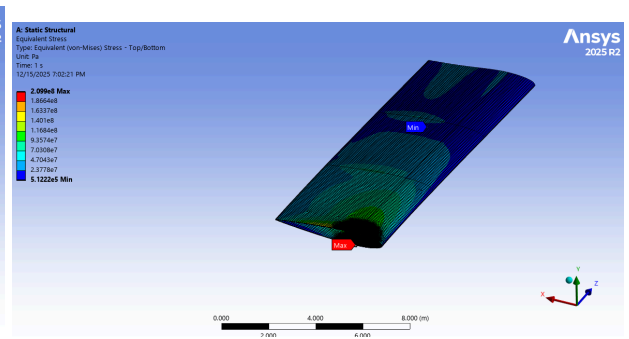
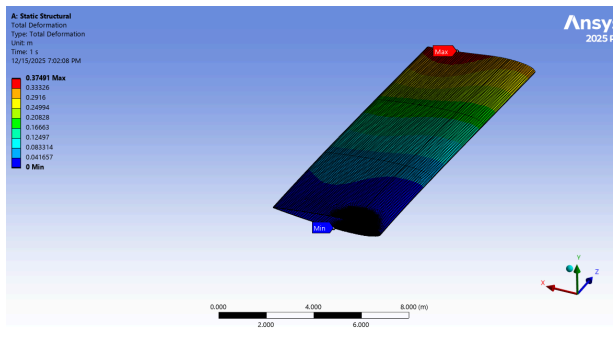
Geometric Variations for Wing Blade Design			
Design #	Major Design Parameter	Description	Geometry Photo
1	<i>Initial (1 Rib, 2 Spar)</i>	The following design is the initial geometric location and orientation for the ribs and spars provided by the course staff. The computations are a reference point for weight improvement.	
2	<i>2 Rib, 1 Spar</i>	This approach reduces the number of spars to determine the overall effect on the aerodynamic load and evaluate methods for reducing weight.	
3	<i>2 Rib, 2 Spar</i>	By increasing the number of ribs, this may assist in distributing aerodynamic load on the blade design. However, this may require additional thickness reduction to prevent excess weight.	
4	<i>15 Rib, 2 Spar</i>	This approach reduces the number of spars to determine the overall effect on the aerodynamic load and evaluate methods for reducing weight. Design informed by general recommendations for a rib every 1-2 meters of the wing blade span.	

Optimized Parameters By Design Variety						
Design #	$t_{Spar}$	$t_{Rib}$	$t_{Skin}$	$\eta_{Min}$	$\delta_{Tip, Max}$	$W_{Est.}$
1A	0.020022 m	0.002501 m	0.02373 m	2.1414	0.37599 m	11039 kg

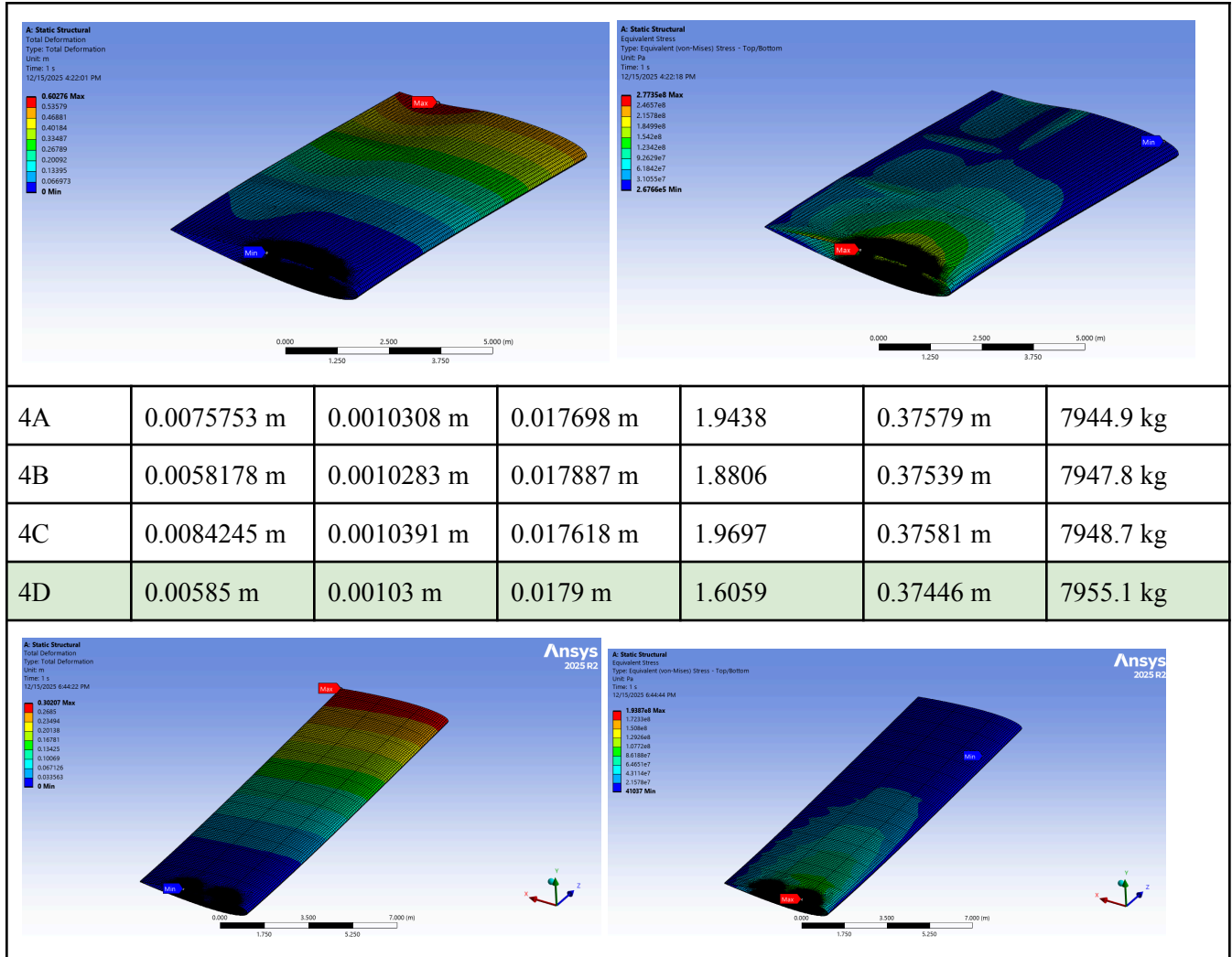
1B	0.20058 m	0.02372 m	0.02372 m	2.1427	0.37599 m	11039
1C	0.02008 m	0.02372 m	0.023721 m	2.1429	0.37599 m	11040 kg
1D	0.01204 m	0.0275 m	0.0188 m	1.9381	0.44145 m	9553.5 kg



2A	0.018821 m	0.012035 m	0.027615 m	2.2733	0.376 m	12345 kg
2B	0.018785 m	0.012109 m	0.027618 m	2.273	0.376 m	12346 kg
2C	0.018714 m	0.012071 m	0.027624 m	2.2727	0.376 m	12346 kg
2D	0.01875 m	0.012000 m	0.027500 m	2.2727	0.37491 m	12380 kg



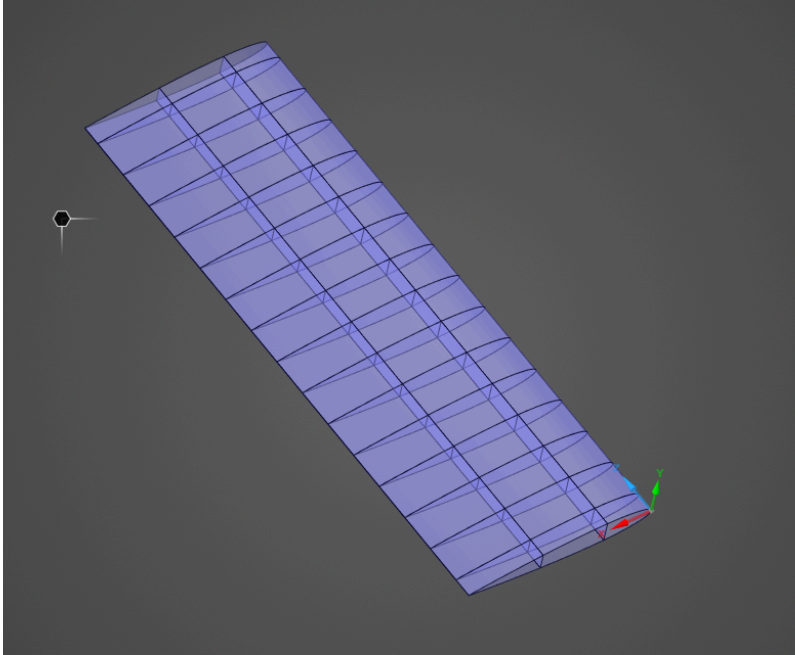
3A	0.014724 m	0.0013135 m	0.023737 m	1.9082	0.37597 m	10797 kg
3B	0.015091 m	0.0013475 m	0.023705 m	1.9111	0.37581 m	10800 kg
3C	0.014888 m	0.0021546 m	0.023717 m	1.9098	0.37599 m	10805 kg
3D	0.015000 m	0.015000 m	0.015000 m	1.3629	0.60276 m	7250.7 kg

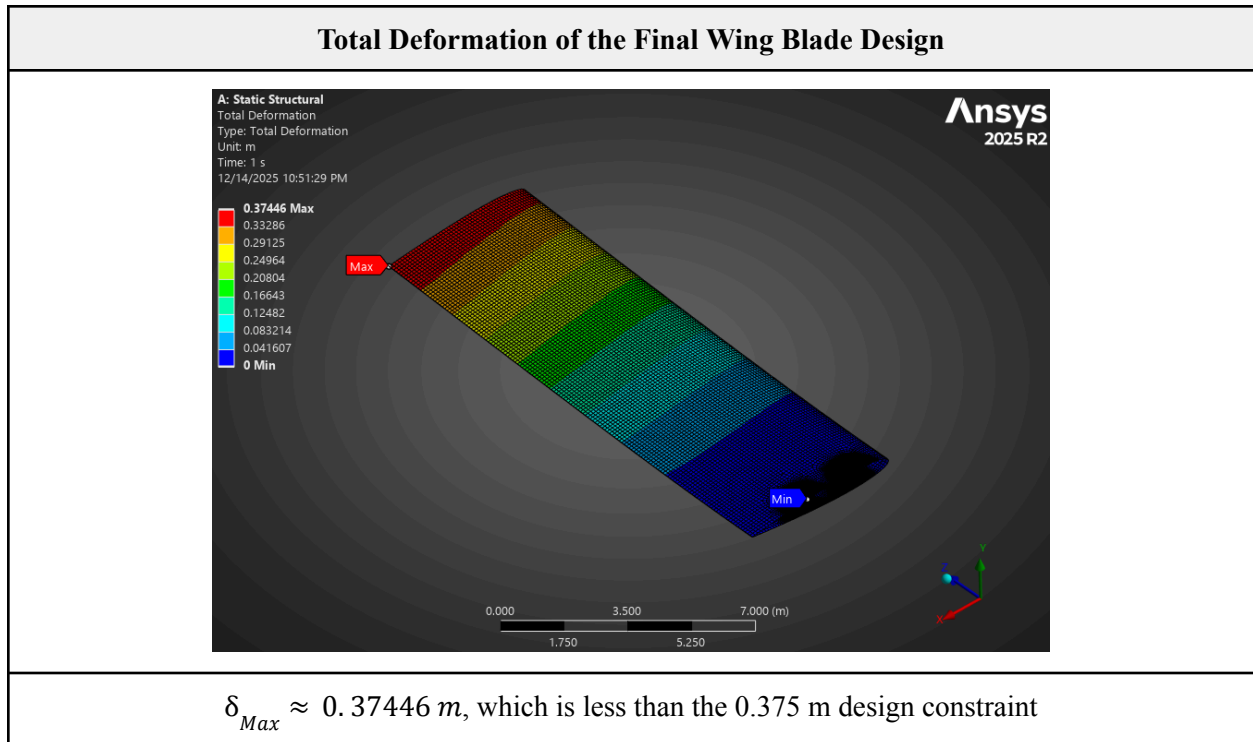


During the optimization process for the wing blade design, the three major constraints were to minimize geometry mass, ensure the factor of safety (FOS) was above 1.5, and to maintain deformation less than 0.375 m. The setup yielded candidate points, which were then further optimized by hand to determine a feasible design given the specific geometric variation. After converging on a specific optimal point for the design, the deformation and stress figures were provided to demonstrate the specific response. Primarily, as the number of ribs increased, there was less material needed to reinforce the wing to meet the FOS requirements, and the weight could be dropped. Additionally, another spar greatly increased the maximum stress of the wing blade, thus leading to our final design integrating an additional spar. After evaluating numerous design points, our group converged on a geometric design that incorporated 15 ribs and 2 spars that were optimized for FOS and stress within the design constraints.

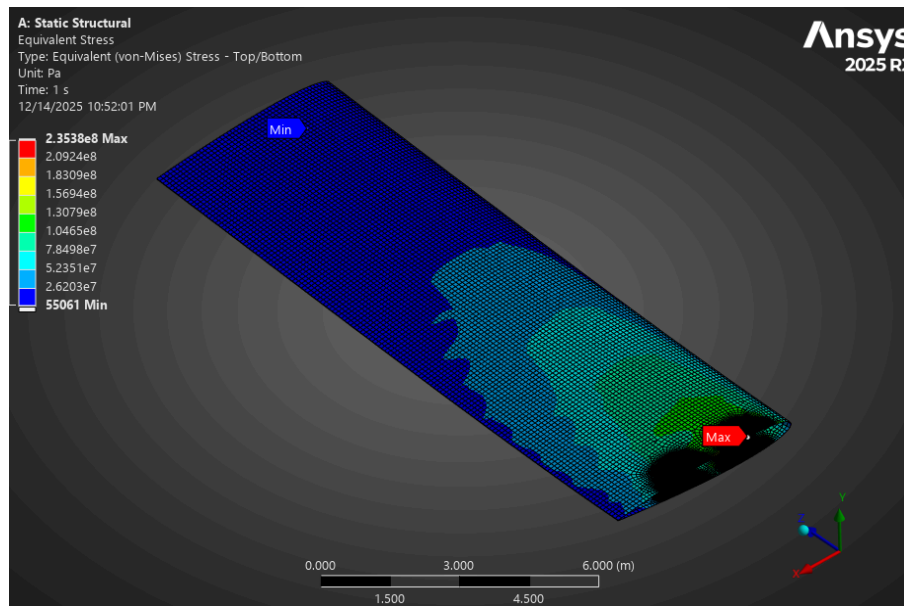
## Recommendations and Improvements

### F. Recommended Design

Wing Design Major Design Attributes	
	<i>Number of Components</i>
	2 Spars
	15 Ribs
	<i>Thickness of Components</i>
	$t_{Spar} = 0.00585 \text{ m}$
	$t_{Rib} = 0.00103 \text{ m}$
	$t_{Skin} = 0.01790 \text{ m}$
<i>Total Weight</i>	
7955.1 kg	



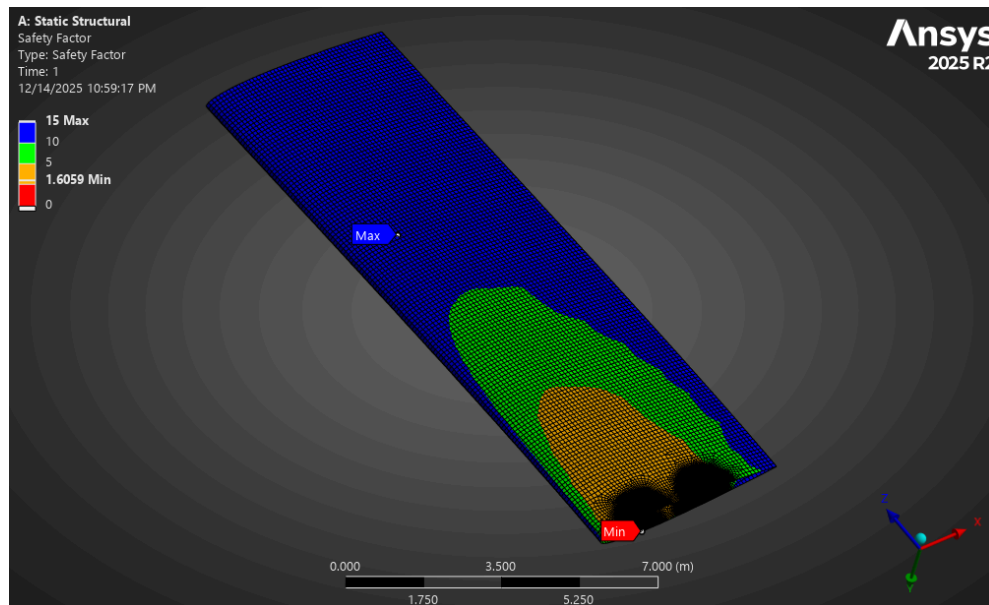
### Total Stress of the Final Wing Blade Design



$$\sigma_{\text{yield}} \approx 235.38 \text{ MPa}$$

$$\sigma_V = \frac{193.87 \text{ MPa}}{1.5} = 156.92 \text{ MPa}, \text{ which is less than the } 378 \text{ MPa design constraint}$$

### Safety Factor of the Final Wing Blade Design



$$\eta \approx 1.6059, \text{ which is higher than the lower end } 1.5 \text{ safety factor design constraint}$$

## G. Conclusion and Future Work

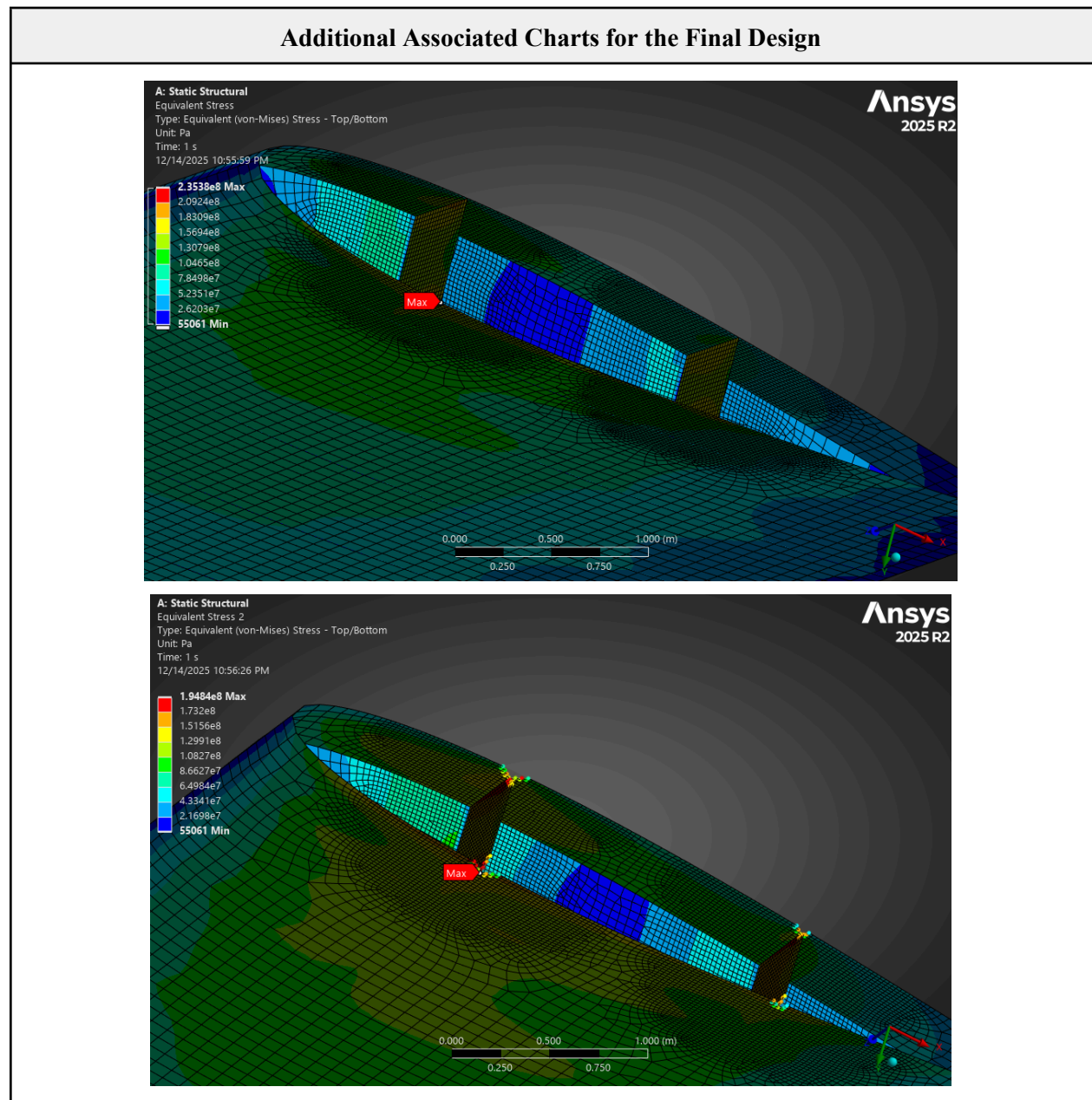
Overall, the finite-element analysis proved successful in improving the overall performance of the original wing blade design by reducing the blade's total weight by 41.80% while remaining within design constraints. Throughout the iterations, major improvements revolved around increasing the number of ribs while reducing the overall thickness of the design. Technically, it is possible to continue optimizing the blade geometry with a higher number of ribs and reduced thickness, but the final blade design's thickness appears to be a realistic floor value. 1/16th (1.29 mm) and 1/32nd (1.02 mm) are common thicknesses for consumer-grade aluminum sheets, and exceeding this ~1.00 mm threshold would certainly be unrealistic for a structural use case. This order of magnitude both grounds the current estimation and further strengthens the case for this blade design.

Although the current approach was a sound and effective measure for evaluating the efficacy of a single-design parameter with finite-element analysis, there are likely more effective structural methods for generalizing the response from the start to contextualize our approach. Online resources indicate that typical airfoils require a spar every 1-2 meters of span, thus confirming the finite-element analysis for the aforementioned wing blade. The major requirement for these computational methods is to first create a strong conceptual foundation for the model, create an inferred hypothesis/ well-educated design for further improvements with hand-calculated structural analysis, and compute in ANSYS. Otherwise, the practice becomes computationally inefficient and redundant.

Future improvements may involve surveying other material types, changing the span of the overall wing blade, or implementing non-constant chord cross-sections. By evaluating the use case of the blade, there is a potential for other materials or composites to be implemented, such as carbon fiber, to reduce the density and mass of the overall system. Additionally, from MAE 4272, experimental testing has proved that mechanical stress due to bending can be distributed through non-constant linear airfoil cross-sections, thus expanding the opportunity to test different geometric parameters as radial functions.

## H. Appendix

Group Contributions	
Andrew D’Onofrio	Writing for the Abstract, Introduction, Mesh Refinement, Mesh Sensitivity; Computations for Design Comparison; Collaborated on Conclusion and Future Work
Chris Sawicki	Computations for Design Comparisons and final Recommended Design; Collaborated on Conclusion and Future Work



# Report Design #1

## Design Points

Name	P7 - Spar Thickness	P13 - Spar2 Thickness	P14 - Spar1 Thickness	P15 - Rib Thickness	P16 - Skin Thickness	P4 - Geometry Mass	P5 - Total Deformation Maximum	P6 - Safety Factor Minimum	Retained	Retained Data State	Report	Note
Units	m	m	m	m	m	kg	m					
DP 0 (Current)	20	0.02	0.02	0.02	0.02	9553.5	0.44145	1.9381	yes	✓		

## Outline of All Parameters

ID	Parameter Name	Value	Unit
<b>Input Parameters</b>			
Static Structural			
P13	Spar2 Thickness	0.02	m
P14	Spar1 Thickness	0.02	m
P15	Rib Thickness	0.02	m
P16	Skin Thickness	0.02	m
P7	Spar Thickness	20	
<b>Output Parameters</b>			
Static Structural			
P4	Geometry Mass	9553.5	kg
P5	Total Deformation Maximum	0.44145	m
P6	Safety Factor Minimum	1.9381	

## Response Surface Optimization

### Design of Experiments

The Design of Experiments is the initial step building a Response Surface over the design space. This section describes the selected input parameters and their variation range, the chosen Design of Experiments type, and the generated Matrix of Experiments.

### Parameters

The explored design space is defined by the range of variation and the allowed values of the following 3 input parameters.

ID	Name	Classification	Lower Bound	Upper Bound	Allowed Values
P7	Spar Thickness	Continuous	20	22.5	Any
P15	Rib Thickness	Continuous	0.0025 [m]	0.0035 [m]	Any
P16	Skin Thickness	Continuous	0.0225 [m]	0.024 [m]	Any

### Design of Experiments Properties

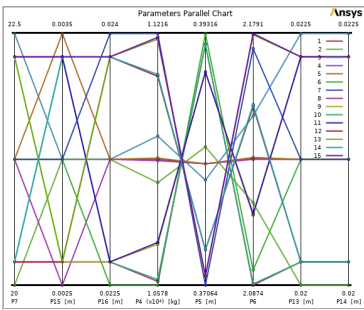
Property	Value
Design of Experiments Type	Central Composite Design
Design Type	Auto Defined

### Matrix of Experiments

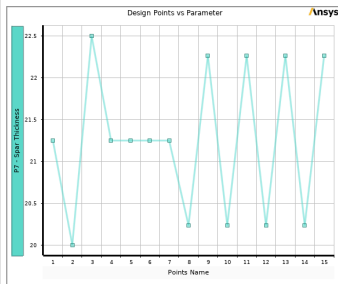
All of the 15 points are up-to-date.

[The full Matrix of Experiments is provided in the Appendices.](#)

### Parameters Parallel Chart



### Design Points vs Parameter Chart



## Response Surface

The Response Surface is a meta-model built from the Design of Experiments for an efficient exploration of the design space. This section describes the selected type of meta-model, including its properties, the obtained quality, and the generated Response Points and charts.

### Response Surface Properties

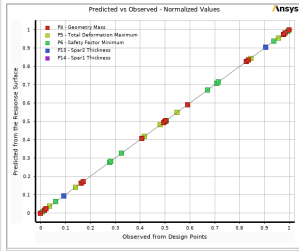
Property	Value
Response Surface Type	Genetic Aggregation
Random Generator Seed	0
Maximum Number of Generations	12
Display Level	Final
Generate Verification Points	No

Goodness of Fit

The Goodness of Fit report provides charts and metrics to understand how each output parameter is approximated by the response surface.

Details of 'Goodness Of Fit'

Predicted vs Observed Scatter Chart



	P4 - Geometry Mass	P5 - Total Deformation Maximum	P6 - Safety Factor Minimum
<b>Coefficient of Determination (Best Value = 1)</b>			
Learning Points	☆☆ 1	☆☆ 1	☆☆ 1
Cross-Validation on Learning Points	☆☆ 1	☆☆ 1	☆☆ 1
<b>Root Mean Square Error (Best Value = 0)</b>			
Learning Points	8.1348E-13	5.9349E-11	2.2561E-06
Cross-Validation on Learning Points	1.1504E-12	1.8571E-06	7.3326E-06
<b>Relative Maximum Absolute Error (Best Value = 0%)</b>			
Learning Points	☆☆ 0	☆☆ 0	☆☆ 0.012491
Cross-Validation on Learning Points	☆☆ 7.7757E-13	☆☆ 0.060018	☆☆ 0.050884
<b>Relative Average Absolute Error (Best Value = 0%)</b>			
Learning Points	☆☆ 0	☆☆ 0	☆☆ 0
Cross-Validation on Learning Points	☆☆ 3.1103E-13	☆☆ 0.013023	☆☆ 0.017389

Minimum and maximum values

This section reports the minimum and maximum values for each output parameter. These values are approximations found by the Min-Max Search on the Response Surface.

Name	Minimum value	Maximum value
P4 - Geometry Mass (kg)	10517	11277
P5 - Total Deformation Maximum (m)	0.36924	0.39472
P6 - Safety Factor Minimum	2.0769	2.1896
P13 - Spar2 Thickness (m)	0.02	0.0225
P14 - Spar1 Thickness (m)	0.02	0.0225

The input parameter values corresponding to the minimum and maximum values are provided in the Appendices.

## Report Design #2

Design Points

Name	P8 - Rib Thickness	P4 - Geometry Mass	P5 - Total Deformation Maximum	P6 - Safety Factor Minimum	Retained	Retained Data State	Report	Note
Units	kg	m						
DP (Current)	20	12380	0.37491	2.2787	yes	✓		

Outline of All Parameters

Parameter Name	Value	Unit
<b>Input Parameters</b>		
P8 - Rib Thickness	20	
<b>Output Parameters</b>		
<b>Static Structural</b>		
p2 P4 - Geometry Mass	12380	kg
p2 P5 - Total Deformation Maximum	0.37491	m
p2 P6 - Safety Factor Minimum	2.2787	

Response Surface Optimization

Design of Experiments

The Design of Experiments is the initial step building a Response Surface over the design space. This section describes the selected input parameters and their variation range, the chosen Design of Experiments type, and the generated Matrix of Experiments.

Parameters

The explored design space is defined by the range of variation and the allowed values of the following 3 input parameters.

ID	Name	Classification	Lower Bound	Upper Bound	Allowed Values
P8	Rib Thickness	Continuous	12	21	Any
P9	Skin Thickness	Continuous	0.025 [m]	0.029 [m]	Any
P12	Spar Thickness	Continuous	0.015 [m]	0.019 [m]	Any

Response Surface Optimization

Design of Experiments

The Design of Experiments is the initial step building a Response Surface over the design space. This section describes the selected input parameters and their variation range, the chosen Design of Experiments type, and the generated Matrix of Experiments.

Parameters

The explored design space is defined by the range of variation and the allowed values of the following 3 input parameters.

ID	Name	Classification	Lower Bound	Upper Bound	Allowed Values
P8	Rib Thickness	Continuous	12	21	Any
P9	Skin Thickness	Continuous	0.025 [m]	0.029 [m]	Any
P12	Spar Thickness	Continuous	0.015 [m]	0.019 [m]	Any

Design of Experiments Properties

Property	Value
Design of Experiments Type	Central Composite Design
Design Type	Auto Defined

Matrix of Experiments

All of the 15 points are up-to-date.

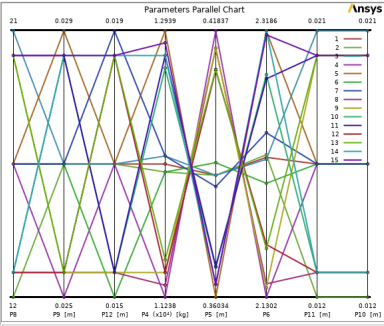
The full Matrix of Experiments is provided in the Appendices.

**Matrix of Experiments**

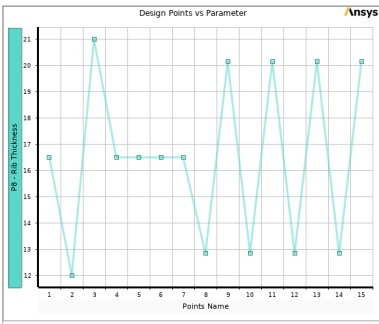
All of the 15 points are up-to-date.

[The full Matrix of Experiments is provided in the Appendices.](#)

**Parameters Parallel Chart**



**Design Points vs Parameter Chart**



**Response Surface**

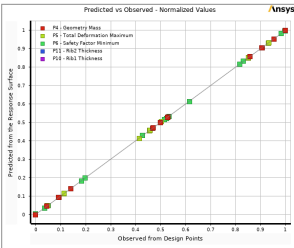
The Response Surface is a meta-model built from the Design of Experiments for an efficient exploration of the design space. This section describes the selected type of meta-model, including its properties, the obtained quality, and the generated Response Points and charts.

**Response Surface Properties**

Property	Value
Response Surface Type	Genetic Aggregation
Random Generator Seed	0
Maximum Number of Generations	12

**Details of 'Goodness of Fit'**

**Predicted vs Observed Scatter Chart**



	P4 - Geometry Mass	P5 - Total Deformation Maximum	P6 - Safety Factor Minimum
<b>Coefficient of Determination (Best Value = 1)</b>			
Learning Points	★ 1	★ 1	★ 0.9999
Cross-Validation on Learning Points	★ 1	★ 1	★ 0.9991
<b>Root Mean Square Error (Best Value = 0)</b>			
Learning Points	1.0920E-12	4.7487E-07	0.0000884
Cross-Validation on Learning Points	3.1894E-12	1.7039E-06	0.0011185
<b>Relative Maximum Absolute Error (Best Value = 0%)</b>			
Learning Points	★ 0	★ 0	★ 0.8867
Cross-Validation on Learning Points	★ 1.1817E-12	★ 0.01661	★ 5.0604
<b>Relative Average Absolute Error (Best Value = 0%)</b>			
Learning Points	★ 0	★ 0	★ 0.28104
Cross-Validation on Learning Points	★ 3.5451E-13	★ 0.003471	★ 1.0956

**Minimum and maximum values**

This section reports the minimum and maximum values for each output parameter. These values are approximations found by the Min-Max Search on the Response Surface.

Name	Minimum value	Maximum value
P4 - Geometry Mass (kg)	11138	13040
P5 - Total Deformation Maximum (m)	0.35821	0.42148
P6 - Safety Factor Minimum	2.115	2.3409
P11 - Rib2 Thickness (m)	0.012	0.021
P10 - Rib1 Thickness (m)	0.012	0.021

[The input parameter values corresponding to the minimum and maximum values are provided in the Appendices.](#)

**Candidates**

Candidates of the optimization study

	P8 - Rib Thickness	P9 - Skin Thickness (m)	P12 - Spar Thickness (m)	P4 - Geometry Mass (kg)	P5 - Total Deformation Maximum (m)	P6 - Safety Factor Minimum	P11 - Rib2 Thickness (m)	P10 - Rib1 Thickness (m)	Report
Candidate Point 1	12.035	0.027615	0.018821	✘ 12345	★ 0.376	★ 2.2733	0.012035	0.012035	
Candidate Point 2	12.109	0.027618	0.018785	✘ 12346	★ 0.376	★ 2.273	0.012109	0.012109	
Candidate Point 3	12.071	0.027624	0.018714	✘ 12346	★ 0.376	★ 2.2727	0.012071	0.012071	
Custom Candidate Point	12	0.0277	0.0188	✘ 12380	★ 0.37488	★ 2.2769	0.012	0.012	

[The table of Rating Values is provided in the Appendices.](#)

**History Chart**

## Report Design #3

**Design Points**

Name	P7 - Spar Thickness	P8 - Rib Thickness	P9 - Skin Thickness	P10 - Rib1 Thickness	P11 - Rib2 Thickness	P12 - Spar1 Thickness	P13 - Spar2 Thickness	P4 - Geometry Mass	P5 - Total Deformation Maximum	P6 - Safety Factor Minimum	Retained	Retained Data State	Report	Note
Units			m	m	m	m	m	kg	m					
DP 0 (Current)	15	15	0.015	0.015	0.015	0.015	0.015	7250.7	0.60276	1.3629	yes	✓		
DP 1	10	15	0.015	0.015	0.015	0.01	0.01	7016.8	0.62259	1.3173				
DP 2	17.5	5	0.0075	0.005	0.005	0.0175	0.0175	4064.6	1.4129	0.79854				
DP 3	17.5	15	0.0075	0.015	0.015	0.0175	0.0175	4178.7	1.4136	0.79752				
DP 4	17.5	5	0.015	0.005	0.005	0.0175	0.0175	7553.6	0.59494	1.3859				

**Outline of All Parameters**

ID	Parameter Name	Value	Unit
<b>Input Parameters</b>			
Static Structural			
P9	Skin Thickness	0.015	m
P10	Rib1 Thickness	0.015	m
P11	Rib2 Thickness	0.015	m
P12	Spar1 Thickness	0.015	m
P13	Spar2 Thickness	0.015	m
P7	Spar Thickness	15	
P8	Rib Thickness	15	
<b>Output Parameters</b>			
Static Structural			
P4	Geometry Mass	7250.7	kg
P5	Total Deformation Maximum	0.60276	m
P6	Safety Factor Minimum	1.3629	

**Response Surface Optimization**

**Design of Experiments**

The Design of Experiments is the initial step building a Response Surface over the design space. This section describes the selected input parameters and their variation range, the chosen Design of Experiments type, and the generated Matrix of Experiments.

**Parameters**

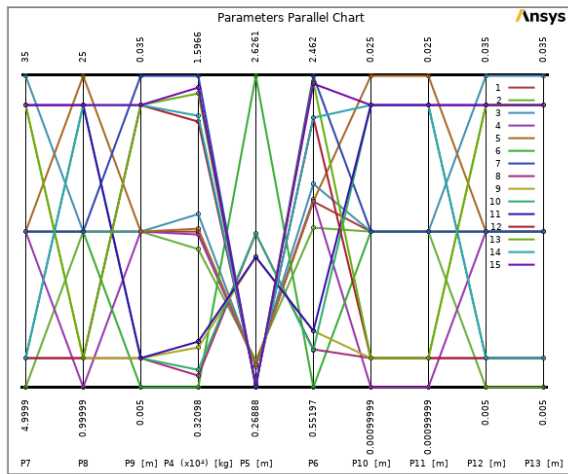
The explored design space is defined by the range of variation and the allowed values of the following 3 input parameters.

ID	Name	Classification	Lower Bound	Upper Bound	Allowed Values
P7	Spar Thickness	Continuous	5	35	Any
P8	Rib Thickness	Continuous	1	35	Any
P9	Skin Thickness	Continuous	0.005 [m]	0.035 [m]	Any

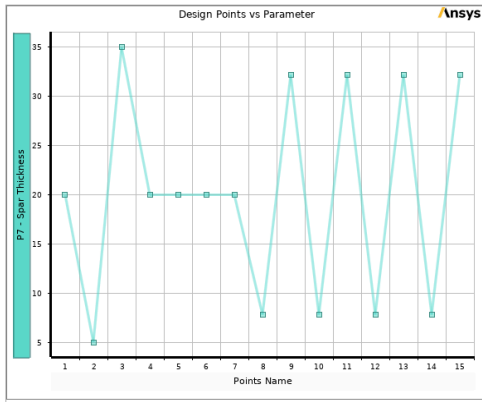
**Design of Experiments Properties**

Property	Value
Design of Experiments Type	Central Composite Design
Design Type	Auto Defined

**Parameters Parallel Chart**



Design Points vs Parameter Chart



Response Surface

The Response Surface is a meta-model built from the Design of Experiments for an efficient exploration of the design space. This section describes the selected type of meta-model, including its properties, the obtained quality, and the generated Response Points and charts.

Response Surface Properties

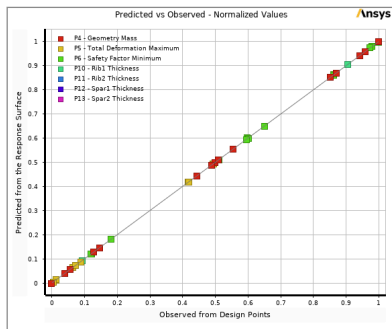
Property	Value
Response Surface Type	Genetic Aggregation
Random Generator Seed	0
Maximum Number of Generations	12
Display Level	Final
Generate Verification Points	No

Goodness of Fit

The Goodness of Fit report provides charts and metrics to understand how each output parameter is approximated by the response surface.

Details of 'Goodness Of Fit'

Predicted vs Observed Scatter Chart



	P4 - Geometry Mass	P5 - Total Deformation Maximum	P6 - Safety Factor Minimum
Coefficient of Determination (Best Value = 1)	☆☆☆☆ 1	☆☆☆☆ 1	☆☆☆☆ 0.99998
Learning Points	☆☆☆☆ 1	☆☆☆☆ 1	☆☆☆☆ 0.99986
Cross Validation on Learning Points	☆☆☆☆ 1	☆☆☆☆ 1	☆☆☆☆ 0.99986
Root Mean Square Error (Best Value = 0)			
Learning Points	1.5669E-12	0.00017054	0.0028823
Cross Validation on Learning Points	4.4089E-12	0.000278	0.0077293
Relative Maximum Absolute Error (Best Value = 0%)			
Learning Points	☆☆ 0	☆☆ 0.05233	☆☆ 1.1674
Cross Validation on Learning Points	☆☆ 2.7498E-13	☆☆ 0.10987	☆☆ 2.4917
Relative Average Absolute Error (Best Value = 0%)			
Learning Points	☆☆ 0	☆☆ 0.017855	☆☆ 0.30135
Cross Validation on Learning Points	☆☆ 6.0233E-14	☆☆ 0.030047	☆☆ 0.86524

Minimum and maximum values

This section reports the minimum and maximum values for each output parameter. These values are approximations found by the Min-Max Search on the Response Surface.

Name	Minimum value	Maximum value
P4 - Geometry Mass (kg)	2371.3	16804
P5 - Total Deformation Maximum (m)	0.26474	3.1681
P6 - Safety Factor Minimum	0.47286	2.5168
P10 - Rib1 Thickness (m)	0.001	0.025
P11 - Rib2 Thickness (m)	0.001	0.025
P12 - Spar1 Thickness (m)	0.005	0.035
P13 - Spar2 Thickness (m)	0.005	0.035

The input parameter values corresponding to the minimum and maximum values are provided in the Appendices.

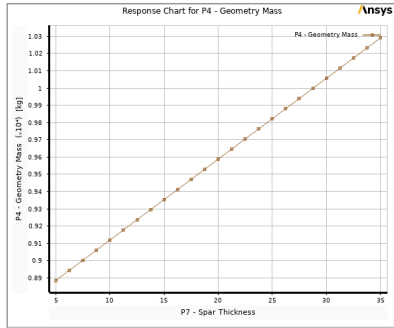
**Response Points**

The Response Points provide the output parameter values obtained by evaluating the Response Surface. This section lists all the Response Points and their associated charts.

Name	P7 - Spar Thickness	P8 - Rib Thickness	P9 - Skin Thickness (m)	P4 - Geometry Mass (kg)	P5 - Total Deformation Maximum (m)	P6 - Safety Factor Minimum	P10 - Rib1 Thickness (m)	P11 - Rib2 Thickness (m)	P12 - Spar1 Thickness (m)	P13 - Spar2 Thickness (m)
Response Point	20	13	0.02	9587.7	0.44088	1.7017	0.013	0.013	0.02	0.02

**Details of 'Response Point'**

**Response Chart**



**Candidates**

Candidates of the optimization study

	P7 - Spar Thickness	P8 - Rib Thickness	P9 - Skin Thickness (m)	P4 - Geometry Mass (kg)	P5 - Total Deformation Maximum (m)	P6 - Safety Factor Minimum	P10 - Rib1 Thickness (m)	P11 - Rib2 Thickness (m)	P12 - Spar1 Thickness (m)	P13 - Spar2 Thickness (m)	Report
Candidate Point 1	14.724	1.3135	0.023737	x 10797	★ 0.37597	★ 1.9082	0.0013135	0.0013135	0.014724	0.014724	
Candidate Point 2	15.091	1.3475	0.023705	x 10800	★ 0.37581	★ 1.9111	0.0013475	0.0013475	0.015091	0.015091	
Candidate Point 3	14.888	2.1546	0.023717	x 10805	★ 0.37599	★ 1.9098	0.0021546	0.0021546	0.014888	0.014888	
Custom Candidate Point	14.73	1	0.0238	x 10820	★ 0.37499	★ 1.912	0.001	0.001	0.01473	0.01473	

[The table of Ratios Values is provided in the Appendices.](#)

**History Chart**

## Report Design #4

**Response Surface Optimization**

**Design of Experiments**

The Design of Experiments is the initial step building a Response Surface over the design space. This section describes the selected input parameters and their variation range, the chosen Design of Experiments type, and the generated Matrix of Experiments.

**Parameters**

The explored design space is defined by the range of variation and the allowed values of the following 3 input parameters.

ID	Name	Classification	Lower Bound	Upper Bound	Allowed Values
P7	Spar Thickness	Continuous	5	35	Any
P16	Skin Thickness	Continuous	0.005 [m]	0.015 [m]	Any
P13	Rib Thickness	Continuous	1	15	Any

**Design of Experiments Properties**

Property	Value
Design of Experiments Type	Central Composite Design
Design Type	Auto Defined

**Matrix of Experiments**

All of the 15 points are up-to-date.

[The full Matrix of Experiments is provided in the Appendices.](#)

**Parameters Parallel Chart**

